On the use of length scale parameters to assess the static strength of notched 3D-printed PLA

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ABSTRACT. This paper aims to investigate the accuracy of the Theory of Critical Distances (TCD) in estimating static strength of notched additively manufactured PLA as both notch sharpness and infill angle vary. The TCD takes as its starting point the assumption that the extent of damage under static loading can be assessed successfully by using two different material parameters, i.e. (i) a critical distance whose length is closely related to the material microstructural features and an inherent (i.e., a defect free) material strength. Plain and notched specimens of 3D-printed PLA were manufactured horizontally by making the deposition angle vary in the range 0°-90°. Using the TCD in the form of the Point Method, failures were predicted by directly post-processing the linear-elastic stress fields estimated through the well-known analytical solutions due to Glinka and Newport. Independently of the notch sharpness, the estimates being obtained were found to be highly accurate, falling within an error interval of about 20%. This result fully supports the idea that the TCD can successfully be used in situations of practical interest to design against static loading notched components of additively manufactured PLA by directly post-processing the results from simple linear-elastic Finite Element (FE) models.

KEYWORDS. Additive Manufacturing; PLA; Theory of Critical Distances; Static Failure; Notches.

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INTRODUCTION

As suggested by ASTM Committee F42 (www.astm.org/COMMITTEE/F42.htm), additive manufacturing is *"the process of joining materials to make objects from 3D-model data, usually layer upon layer, as opposed to subtractive manufacturing methodologies"*. In more detail, additive manufacturing is an "additive" process that can be more effective than

conventional "subtractive" technologies, with this holding true especially in the presence of complex shapes that would be very difficult to be manufactured by using traditional techniques.
If attention is focused specifically on plastics, examination of the state of the art suggests that acrylonitrile butadiene styrene (ABS) and polylactide (PLA) are the two materials that are commonly employed along with low-cost 3D printers, where polymers can be additively manufactured by melting/extruding powders, wires and flat sheets.
As mentioned earlier, the unique feature of additive manufacturing is that parts with complex geometries can be fabricated very easily, with the obtained objects being characterized by a remarkable level of accuracy in terms of both shape and dimensions. This makes it evident that 3D-printed objects can contain very complex geometrical features that are likely to act as local stress concentrators. Therefore, accurate and simple design methods are needed to allow structural engineers to effectively assess the static strength of 3D-printed materials by taking explicitly into account the presence of stress raisers of all kinds.
As far static assessment of notched components is concerned, examination of the state of the art suggests [1-9] that the Theory of Critical Distances (TCD) is the most powerful candidate to be used to design additively manufactured components for the following reasons: (i) the TCD assesses the detrimental effect of notches independently from their shape and sharpness; (ii) it models explicitly the material morphology through *ad hoc* critical lengths; (iii) the required local stress fields can be estimated without adopting complex non-linear constitutive laws; (iv) the TCD can be applied along with the results from linear-elastic FE models, where the same solid models can be used also to inform the manufacturing process.
In this challenging scenario, this paper aims to investigate whether the linear-elastic TCD is successful in performing the static assessment of 3D-printed notched PLA subjected to in service static loading. To conclude, it is worth observing that the present investigation was based on PLA since this material is a biodegradable, absorbable and biocompatible thermoplastic aliphatic polyester that is widely used to manufacture biomedical components having complex shape [10]. Further, thanks to its specific features, PLA is one of those polymers that can be additively manufactured at a relatively low-cost by employing commercial 3D-printers.

STATIC ASSESSMENT ACCORDING TO THE THEORY OF CRITICAL DISTANCES

According to the TCD, the static breakage of notched materials subjected to Mode I loading is avoided as long as a critical distance based effective stress, σ_{eff}, is lower than the material inherent strength, σ_0 [1], i.e.:

$$\sigma_{eff} \leq \sigma_0 \tag{1}$$

An important feature of the TCD is that it assesses static strength by directly post-processing the linear-elastic stress fields damaging the material in the vicinity of the notch being designed, with this holding true independently from the ductility level of the material under investigation [1-4, 6]. This can be done successfully provided that material inherent strength σ_0 is determined consistently [3, 4]. The procedure being recommended to be followed to determine σ_0 experimentally will be reviewed in what follows. These considerations suggest that the TCD is bi-parametrical design method where the critical distance and the inherent material strength are the two material parameters being used.
Under static loading, the TCD's length scale parameter is recommended to be estimated according to this well-known relationship [1, 11]:

$$L = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_0}\right)^2 \tag{2}$$

where K_{Ic} is the plane strain fracture toughness. As soon as material length L is known, the required effective stress, σ_{eff}, can then be calculated directly according to either the Point Method (PM) or the Line Method (LM) as follows [1, 11]:

$$\sigma_{eff} = \sigma_y\left(\theta = 0, r = \frac{L}{2}\right) \qquad \text{Point Method (PM)} \tag{3}$$

$$\sigma_{eff} = \frac{1}{2L}\int_{0}^{2L} \sigma_y(\theta = 0, r)\, dr \qquad \text{Line Method (LM)} \tag{4}$$

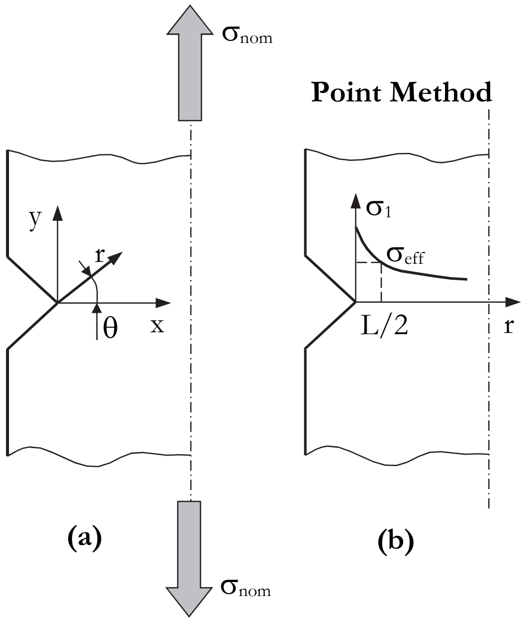

Figure 1: Definition of the local systems of coordinates (a) and effective stress, σ_{eff}, calculated according to the Point Method (b) and the Line Method (c).

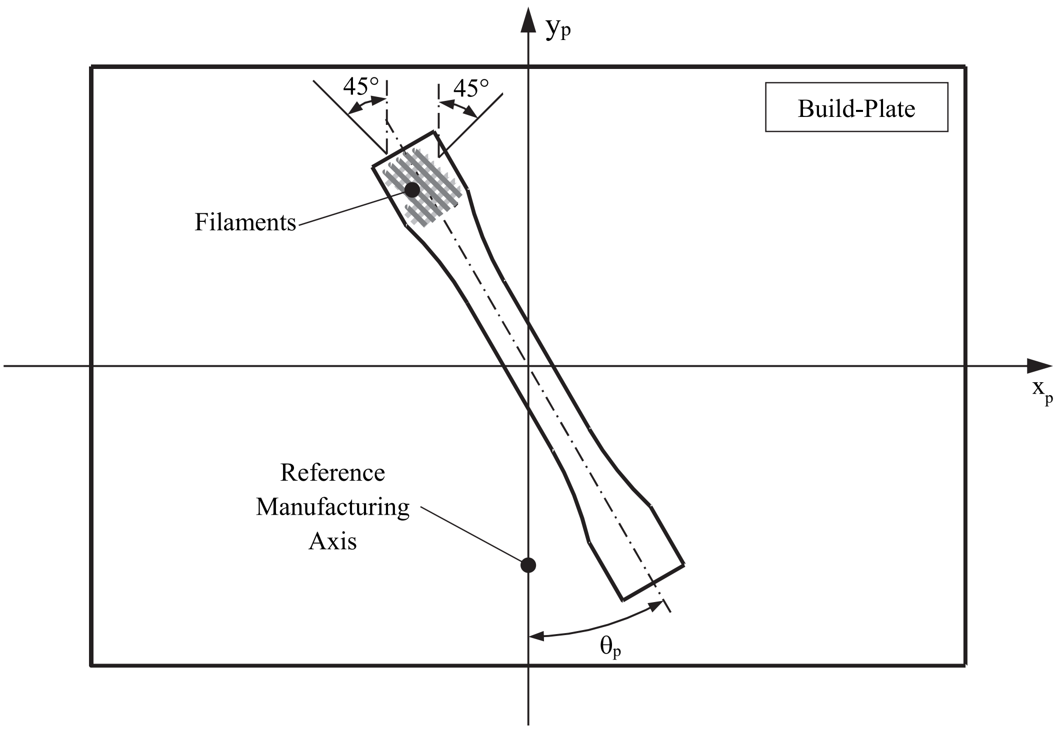

Figure 2: Manufacturing direction and orientation of the deposition filaments.

The adopted symbols as well as the meaning of the effective stress determined according to definitions (3) and (4) are explained in Fig. 1. As to the different formalisations of the TCD, it is worth mentioning here that this powerful theory can be applied also in the form of the Area Method (AM) [1, 12] as well as of the Volume Method (VM) [1, 13]. In more detail, the AM postulates that the effective stress has to be determined by averaging σ_1 over a semi-circular area centred at the notch tip and having radius equal to L [1]. In a similar way, the VM calculates σ_{eff} by averaging the linear-elastic maximum principal stress over a hemisphere centred at the apex of the stress raiser being assessed and having radius equal to 1.54L [12]. In the present investigation, owing to its simplicity, the accuracy of the TCD in estimating static strength of notched additively manufacture PLA will be assessed by applying this theory solely in the form of the PM.

The definitions for σ_{eff} calculated according to Eqs (3) and (4) make it clear that inherent material strength σ_0 plays a role of primary importance when the TCD is used to assess static strength of notched components. As far as brittle materials are concerned, much experimental evidence suggests that these materials have inherent strength that is always very close to the ultimate tensile strength, σ_{UTS} [1-3]. In contrast, when the final breakage is preceded by large scale plastic deformations, σ_0 is seen to be somewhat larger than σ_{UTS} [1, 4, 6]. Further, σ_0 takes on a value that is higher than σ_{UTS} also when the plain parent material fails by different mechanisms to those leading to the final breakage in the presence of stress raisers [1]. These considerations should make it evident that the only way to determine σ_0 accurately is by running bespoke experiments involving notches whose presence results in different stress distributions in the vicinity of the tested geometrical features [1-6].

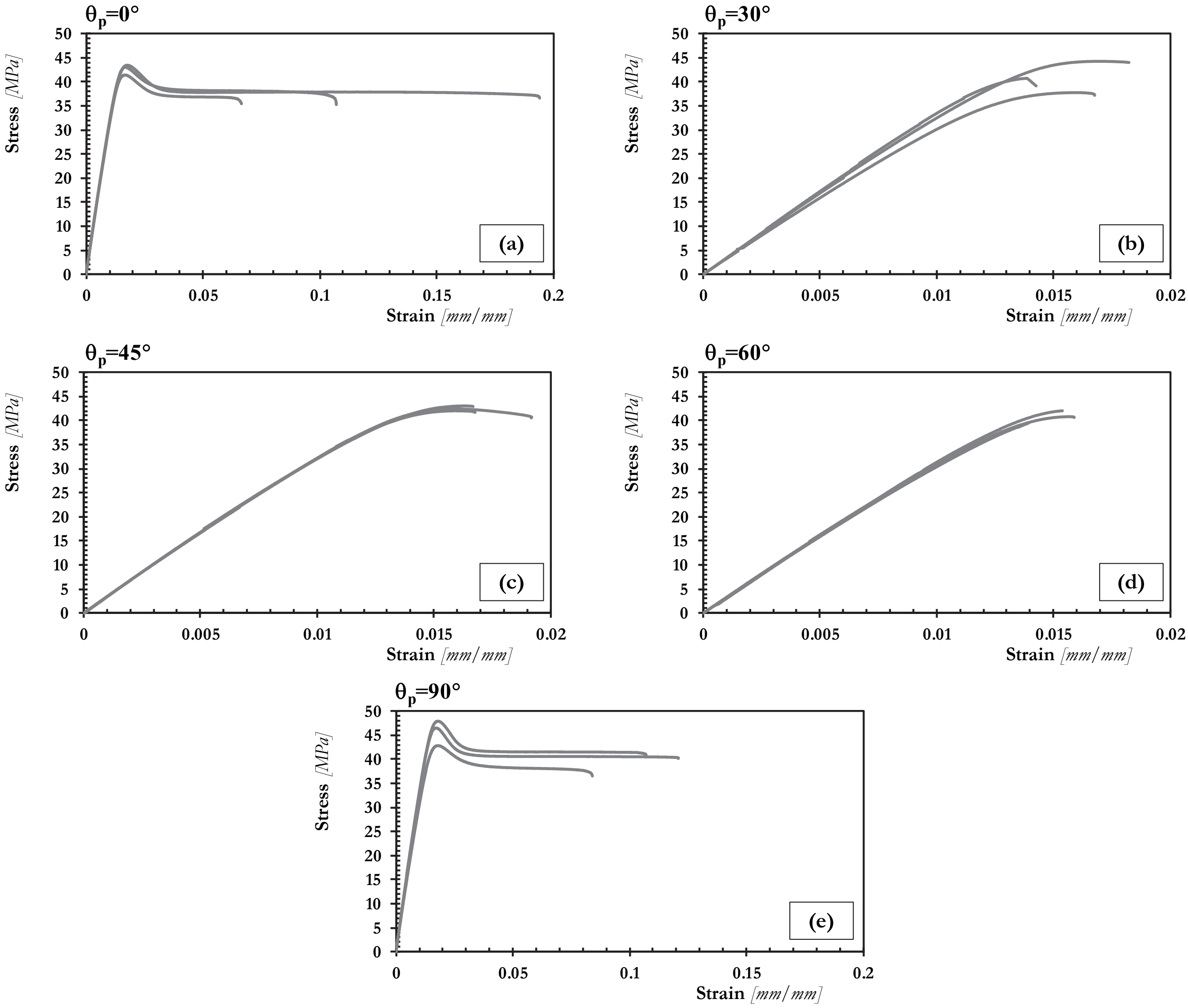

Figure 3: Experimental stress vs. strain curves generated by testing plain samples manufactured by adopting different values for manufacturing angle θ_p.

EXPERIMENTAL RESULTS

Plain and notched specimens were additively manufactured via 3D-printer Ultimaker 2 Extended+ by using as parent material New Verbatim filaments of white PLA with initial diameter of 2.85mm. The values of the adopted manufacturing parameters were as follows: nozzle size=0.4 mm, nozzle temperature=240ºC, build-plate temperature=60ºC, layer height=0.1 mm, shell thickness=0.4 mm, fill density=100%, and print speed=30 mm/s. According to Fig. 2, the flat samples being tested were manufactured horizontally on the build-plate by setting angle θ_p equal to 0º, 30º, 45º, 60º, and 90º. In particular, while the extruded filaments were deposited, layer upon layer, always at ±45º to the axis y_p of the build-plate, the angle between the longitudinal axis of the specimens and axis y_p was varied in the range 0º-90º (see Fig. 2).

All samples had thickness, t, equal to 4mm, with the un-notched specimens having net width equal to 15 mm. The sharply U-notched specimens being manufactured had net width, w_n, equal to 15.4 mm, gross width, w_g, to 24.9 mm, and notch root radius, r_n, equal to 0.5 mm. These dimensions returned a value for the net stress concentrator factor, K_t, equal to 4.76. The specimens containing the intermediate U-notches had the following average dimensions: w_n=15.3 mm, w_g=24.9 mm, and r_n=1.0 mm (resulting in a K_t value equal to 3.51). Finally, the dimensions of the bluntly U-notched specimens were as follows: w_n=15.2 mm, w_g=25.1 mm, and r_n=3.0 mm (K_t=2.22). For the notched specimens being investigated the corresponding values for the stress concentration factors were estimated using Peterson's diagrams [14]. A number of specimens containing crack-like notches were also additively manufactured in order to determine the fracture toughness for t=4 mm. In particular, the relevant dimensions for these specimens were as follows: w_n=16.4 mm, w_g=24.9 mm, and $r_n \approx 0.01$ mm.

By setting the displacement rate equal to 2 mm/min, the plain specimens as well as the samples containing both U-notches and crack-like notches were tested under quasi-static tensile loading by using a Shimadzu universal machine. In the un-notched specimens, the local strains were measured during testing via an axial extensometer with gauge length equal to 50 mm. Three different specimens were tested for any geometry/manufacturing configuration that was investigated.

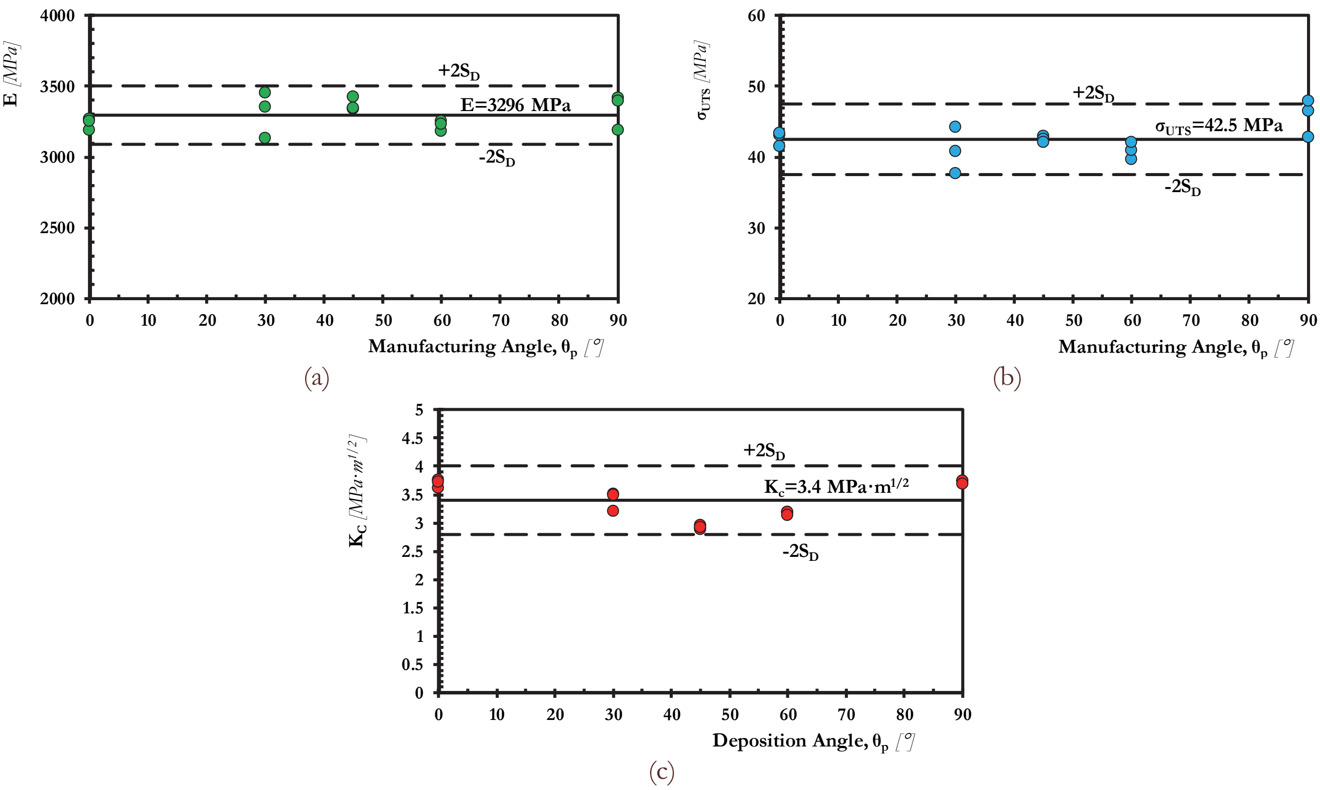

Figure 4: Influence of manufacturing angle θ_p on E (a), σ_{UTS} (b), and K_C (c).

The stress vs. strain diagrams reported in Fig. 3 show the mechanical behaviour as measured by testing the un-notched specimens. These charts make it evident that the stress vs. strain response of the additively manufactured material being tested was characterised by a mechanical behaviour that was predominantly linear up to the maximum stress value being recorded during testing.

In terms of reference mechanical properties, the results generated by testing the plain samples returned the following average values: Young's modulus, E, equal to 3296 MPa, 0.2% proof stress, $\sigma_{0.2\%}$, equal to 40.3 MPa and ultimate tensile strength, σ_{UTS}, equal to 42.5 MPa. The charts reported in Figs. 4a and 4b summarise the influence of manufacturing angle θ_p on both E and σ_{UTS}. These diagrams make it evident that the infill direction had little effect on the overall mechanical behaviour of the additively manufactured material being investigated. In particular, the experimental results expressed in terms of both E and σ_{UTS} in Figs. 4a and 4b, respectively, are seen to be all within two standard deviations, S_D, of the mean.

The results generated by testing specimens with crack-like notches are summarised instead in the K_C (for t=4 mm) vs. θ_p diagram of Fig. 4c. The values for K_C shown in this chart were estimated according to Linear Elastic Fracture Mechanics (LEFM), i.e. by using the following standard relationship [15]:

$$K_C = \alpha \cdot \sigma_f \cdot \sqrt{\pi a} \qquad (5)$$

In Eq. (5) α is the shape factor (estimated as recommended in Ref. [16]), σ_f is the nominal failure stress referred to the gross area, and a is the crack-like notch depth. Since, according to Fig. 4c, the experimental values for K_C are within two standard deviations of the mean, it is possible to come to the conclusion that also the fracture resistance of the additively manufacture polymer under investigation was marginally affected by manufacturing angle θ_p.

To conclude, it is worth observing that, due to such a high level of consistency, the notched specimens being tested were manufactured by setting angle θ_p solely equal to 0°, 30°, and 45°.

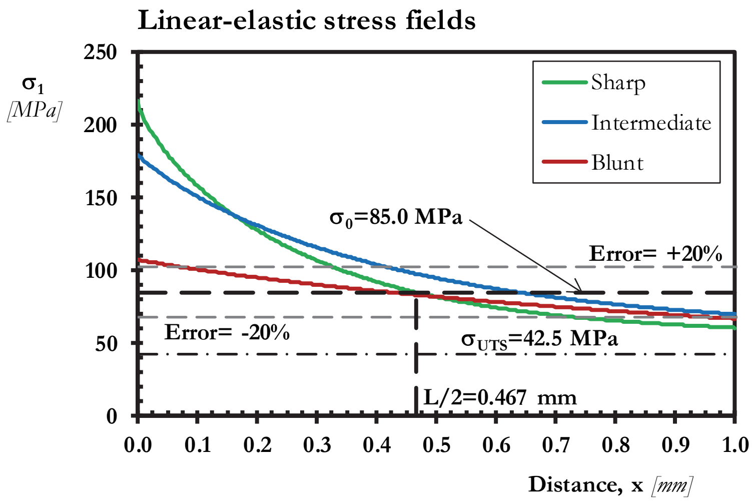

Figure 5: Linear-elastic stress fields in the incipient failure condition and determination of critical distance L.

VALIDATION BY EXPERIMENTAL DATA

By considering notched specimens of polymethylmethacrylate (PMMA), in 2004 Taylor et al. [17] observed that the inherent strength, σ_0, of this material takes on a value that is equal to about $2 \cdot \sigma_{UTS}$. Accordingly, the standard formalisation of the TCD considered in the present paper can be used to perform the static assessment of PMMA as long the notches being assessed result in stress concentration factors that are larger than $\sigma_0/\sigma_{UTS} \approx 2$.

Having recalled these important aspects, in the preliminary investigating summarised in the present paper the accuracy of the TCD in estimating the static strength of notched additively manufactured PLA was checked by forming the following simplifying hypotheses:

- according to Figs. 4b and 4c, the static strength and the fracture toughness of the additively manufactured material being tested were assumed to be independent from manufacturing angle θ_p;
- as per PMMA [17], the inherent strength, σ_0, of the additively manufactured material under investigation was taken invariably equal to $2 \cdot \sigma_{UTS}$ – i.e., $\sigma_0 = 2 \cdot \sigma_{UTS} = 2 \cdot 42.5$ MPa=85 MPa;
- the linear-elastic stress fields in the vicinity of the stress concentrators being tested were estimated by simply using the analytical solutions devised by Glinka and Newport [18].

Fig. 5 shows the linear-elastic stress fields determined in the incipient failure condition, where the centre of the adopted system of coordinates was taken coincident with the notch tip (Fig. 1a). It is worth observing here also that, for any considered notched geometry, the corresponding stress field was calculated by setting the nominal net stress equal to the average failure stress determined from the nine tests run by using the specimens manufactured with angle θ_p equal to 0º, 30º, and 45º (Fig. 2).

As shown in Fig. 5, the critical distance value was estimated according to the PM via the point at which the linear-elastic stress-distance curve due to the sharp notch and the horizontal straight line modelling the plain material inherent strength intersected each other [1]. According to Fig. 5, this simple procedure returned a value for L of 0.934 mm. Fig. 5 makes it evident also that by setting L/2=0.467 mm and σ_0=85 MPa, the PM was seen to be very accurate also in estimating the average static strength of the specimens containing both the intermediate and the blunt notches.

In order to show the overall accuracy of the PM in performing the static assessment of notched additively manufactured PLA, the chart of Fig. 6 summarises the error associated with the individual experimental results, with the error being calculated as:

$$Error\ [\%] = \frac{\sigma_{eff} - \sigma_0}{\sigma_0} \cdot 100 \qquad (6)$$

According to the error diagram reported in Fig. 6, it is possible to conclude by observing that the TCD applied in the form of the PM was seen to be capable of accurately estimating the static strength of notched additively manufactured PLA, with its usage returning predictions falling within an error interval of ±20%.

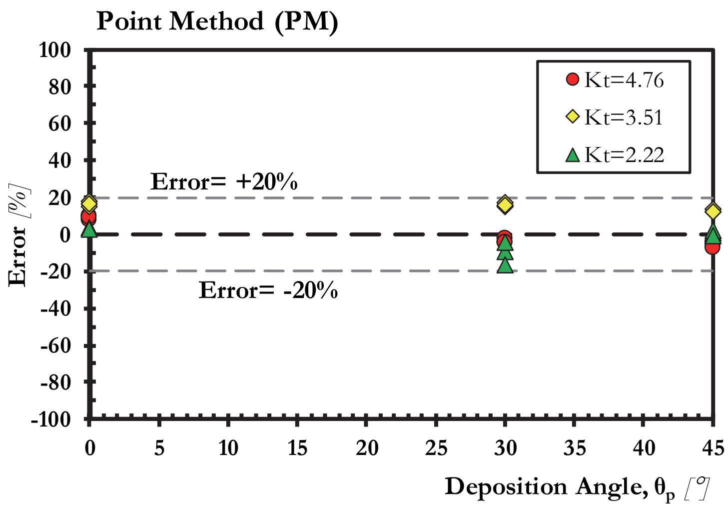

Figure 6: Overall accuracy of the TCD applied in the form of the PM in estimating static strength of additively manufactured PLA.

CONCLUSIONS

According to the analyses summarised in the present paper, the following conclusions can be drawn:

- the TCD is successful in predicting static failures in notched additively manufactured PLA;
- the estimates obtained by applying the TCD in the form of the PM were seen to fall within an error interval of ±20%;

- satisfactory results were obtained by forming the hypothesis that the mechanical behaviour of additively manufactured PLA follows a simple linear-elastic constitutive law;
- more work has to be done in this area to systematically check the accuracy and reliability of the TCD when used to predict static failures in additively manufactured materials not only characterised by different elasto-plastic behaviours, but also failing by different mechanisms.

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